Q 2.1) Two charges 5 × 10^{-8} C and -3 x 10^{-8} C are the two charges located 16 cm apart from each other. At what point(s) on the line joining the two charges is the electric potential zero? Take the potential at infinity to be zero.

Soln.: Given,

q_1 = 5 x 10^{-8} C

q_2 = -3 x 10^{-8} C

The two charges are at a distance, d = 16cm = 0.16m from each other.

As shown in the figure, let us consider a point P over the line joining charges q_1 and q_2.

Let, distance of the considered point P from q_1 be 'r'

Let, point P has zero electric potential (V).

The electric potential at point P is the summation of potentials due to charges q_1 and q_2.

Therefore, $V = \frac{1}{4\pi\epsilon_o}\cdot\frac{q_1}{r} + \frac{1}{4\pi\epsilon_o}\cdot\frac{q_2}{d-r}$ **..........(1)**

Here,

ϵ_o = permittivity of free space.

Putting V = 0, in eqn. (1), we get,

0 = $\frac{1}{4\pi\epsilon_o}\cdot\frac{q_1}{r} + \frac{1}{4\pi\epsilon_o}\cdot\frac{q_2}{d-r}$ $\frac{1}{4\pi\epsilon_o}\cdot\frac{q_1}{r} = -\frac{1}{4\pi\epsilon_o}\cdot\frac{q_2}{d-r}$ $\frac{q_1}{r} = -\frac{q_2}{d-r}$ $\frac{5\times10^{-8}}{r} = -\frac{(-3\times10^{-8})}{0.16-r}$

5(0.16 – r) = 3r

0.8 = 8r

r = 0.1m = 10 cm.

Therefore, at a distance of 10 cm from the positive charge the potential is zero between the two charges.

Let us assume a point P at a distance 's' from the negative charge be outside the system, having potential zero.

So, for the above condition, potential is given by –

$V = \frac{1}{4\pi\epsilon_o}\cdot\frac{q_1}{s} + \frac{1}{4\pi\epsilon_o}\cdot\frac{q_2}{s-d}$ ....(2)

Here,

ϵ_o = permittivity of free space.

For V = 0, eqn. (2) can be written as :

0 = $\frac{1}{4\pi\epsilon_o}\cdot\frac{q_1}{s} + \frac{1}{4\pi\epsilon_o}\cdot\frac{q_2}{s-d}$ $\frac{1}{4\pi\epsilon_o}\cdot\frac{q_1}{s} = -\frac{1}{4\pi\epsilon_o}\cdot\frac{q_2}{s-d}$ $\frac{q_1}{s} = -\frac{q_2}{s-d}$ $\frac{5\times10^{-8}}{s} = -\frac{(-3\times10^{-8})}{s-0.16}$

5(s – 0.16) = 3s

0.8 = 2s

S = 0.4 m = 40 cm.

Therefore, at a distance of 40 cm from the positive charge outside the system of charges, the potential is zero.

Q 2.2) A regular hexagon of side 10 cm has a charge 5 µC at each of its vertices. Calculate the potential at the centre of the hexagon.

Soln.:

The figure shows, regular hexagon containing charges q, at each of its vertices.

Here

q = 5 μC = 5 x 10^{-6} C.

Length of each side of hexagon, AB =BC = CD = DE = EF = FA = 10 cm.

Distance of the vertices from the centre O, d = 10 cm.

Electric potential at point O,

V = $\frac{1}{4\pi\epsilon_o} \cdot \frac{6xq}{d}$

Here,

ϵ_o = Permittivity of free space and $\frac{1}{4\pi\epsilon_o}$ = 9 x 10^9 Nm^2C^{-2}

V = $\frac{9x10^9\ x6x5x10_{-6}}{0.1}$ = 2.7 x 10^6 V.

Q 2.3) Two charges 2 μC and -2 μC are placed at points A and B 6 cm apart.

(1) Identify an equipotential surface of the system.

(2) What is the direction of the electric field at every point on this surface?

Soln.:

(1) An equipotential surface is defined as the surface over which the total potential is zero. In the given question this plane is normal to line AB. The plane is located at the mid – point of the line AB as the magnitude of the charges are same.

(2) At every point on this surface the direction of the electric field is normal to the plane in the direction of AB.

Q 2.4) A spherical conductor of radius 12 cm has a charge of 1.6 x 10^{-7}C distributed uniformly on its surface. What is the electric field

(1) inside the sphere.

(2) just outside the sphere.

(3) at a point 18 cm from the centre of sphere.

Soln.:

(1) Given,

Radius of spherical conductor, r = 12cm = 0.12m

Charge is distributed uniformly over the surface, q = 1.6 x 10^{-7} C.

Electric field inside a spherical conductor is zero.

(2) Electric field E, just outside the conductor is given by the relation,

E = $\frac{1}{4\pi\epsilon_o} \cdot \frac{q}{r^2}$

Here,

= permittivity of free space and $\frac{1}{4\pi\epsilon_o}$ = 9 x 10^9 Nm^2C^{-2}

Therefore,

E = $\frac{9\times10^9\times1.6\times10^{-7}}{(0.12)^2}$

Therefore, just outside the sphere the electric field is 4.4 x 10^4 NC^{-1}.

(3) From the centre of sphere the electric field at a point 18m = E_1.

From the centre of sphere the distance of point d = 18 cm = 0.18m.

$E_1 = \frac{1}{4\pi\epsilon_o} \cdot \frac{q}{d^2} = \frac{9\times10^9\times1.6\times10^{-7}}{((1.8\times10)^{-2})^2} = 4.4 \times 10^4\ NC^{-1}$

So, from the centre of sphere the electric field at a point 18 cm away is $4.4 \times 10^4\ NC^{-1}$.

Q 2.5) A parallel plate capacitor with air between the plates has a capacitance of 8pF (1pF = 10^{-12} F. What will be the capacitance if the distance between the plates is reduced by half, and the space between them is filled with a substance of dielectric constant 6?

Soln.: Given,

Capacitance, C = 8pF.

In first case the parallel plates are at a distance 'd' and is filled with air.

Air has dielectric constant, k = 1

Capacitance, $C = \frac{k\times\epsilon_o\times A}{d} = \frac{\epsilon_o\times A}{d}$ … eq(1)

Here,

A = area of each plate

ϵ_o = permittivity of free space.

Now, if the distance between the parallel plates is reduced to half, then $d_1 = d/2$

Given, dielectric constant of the substance, $k_1 = 6$

Hence, the capacitance of the capacitor,

$C_1 = \frac{k_1\times\epsilon_o\times A}{d_1} = \frac{6\epsilon_o\times A}{d/2} = \frac{12\epsilon_o A}{d}$ … (2)

Taking ratios of eqns. (1) and (2), we get,

C_1 = 2 x 6 C = 12 C = 12 x 8 pF = 96pF.

Hence, capacitance between the plates is 96pF.

Q 2.6) Three capacitors connected in series have capacitance of 9pF each.

(1) What is the total capacitance of the combination?

(2) What is the potential difference across each capacitor if the combination is connected to a 120 V supply?

Soln.:

(1) Given,

The capacitance of the three capacitors, C = 9 pF

Equivalent capacitance (c_{eq}) is the capacitance of the combination of the capacitors given by

$\frac{1}{C_{eq}} = \frac{1}{C} + \frac{1}{C} + \frac{1}{C} = \frac{3}{C} = \frac{3}{9} = \frac{1}{3}$ $\frac{1}{C_{eq}} = \frac{1}{3}$ = C_{eq} = 3 pF

Therefore, the total capacitance = 3pF.

(2) Given, supply voltage, V = 100v

The potential difference (V_1) across the capacitors will be equal to one – third of the supply voltage.

Therefore, $V_1 = \frac{V}{3} = \frac{120}{3}$ = 40V

Hence, the potential difference across each capacitor is 40V.

Q 2.7) Three capacitors of capacitances 2 pF, 3 pF and 4 pF are connected in parallel.
(1) What is the total capacitance of the combination?

(2) Determine the charge on each capacitor if the combination is connected to a 100 V supply.

Soln.:

(1) Given, C_1 = 2pF, C_2 = 3pF and C_3 = 4pF.

Equivalent capacitance for the parallel combination is given by C_{eq} .

Therefore, $C_{eq} = C_1 + C_2 + C_3$ = 2 + 3 + 4 = 9pF

Hence, total capacitance of the combination is 9pF.

(2) supply voltage, V = 100v

The three capacitors are having the same voltage, V = 100v

q = vc

where,

q = charge

c = capacitance of the capacitor

v = potential difference

for capacitance, c = 2pF

q = 100 x 2 = 200pC = 2×10^{-10}C

for capacitance, c = 3pF

q = 100 x 3 = 300pC = 3×10^{-10}C

for capacitance, c = 4pF

q = 100 x 4 = 400pC = 4×10^{-10} C

Q 2.8) In a parallel plate capacitor with air between the plates, each plate has an area of 6×10^{-3} m ^{2} and and the distance between the plates is 3 mm. Calculate the capacitance of the capacitor. If this capacitor is connected to a 100 V supply, what is the charge on each plate of the capacitor?

Soln.: Given,

The area of plate of the capacitor, A = 6×10^{-3} m^2

Distances between the plates, d = 3mm = 3×10^{-3} m

Voltage supplied, V = 100v

Capacitance of a parallel plate capacitor is given by, C = $\frac{\epsilon \times A}{d}$

Here,

ε = permittivity of free space = 8.854×10^{-12} N^{-1} m^{-2} C^{-2}

C = $\frac{8.854\times10^{-12}\times6\times10^{-3}}{3\times10^{-3}}$ = 17.81×10^{-12} F = 17.71 pF.

Therefore, each plate of the capacitor is having a charge of

q = VC = 100 × 17.81×10^{-12} C = 1.771×10^{-9} C

Q 2.9: Explain what would happen if, in the capacitor given in Exercise 2.8, a 3 mm thick mica sheet (of dielectric constant = 6) were inserted between the plates,

(a)while the voltage supply remained connected.

(b)after the supply was disconnected.

Answer 2.9:

(a) Dielectric constant of the mica sheet, k = 6

If voltage supply remained connected, voltage between two plates will be constant.

Supply voltage, V = 100 V

Initial capacitance, C = 1.771×10^{-11} F

New capacitance, $C_1 = kC = 6 \times 1.771 \times 10^{-11}$ F = 106 pF

New charge, $q_1 = C_1V = 106 \times 100$ pC $= 1.06 \times 10^{-8}$ C

Potential across the plates remains 100 V.

(b) Dielectric constant, k = 6

Initial capacitance, $C = 1.771 \times 10^{-11}$ F

New capacitance, $C_1 = kC = 6 \times 1.771 \times 10^{-11}$ F = 106 pF

If supply voltage is removed, then there will be constant amount of charge in the plates.

Charge $= 1.771 \times 10^{-9}$ C

Potential across the plates is given by,

$$V1 = q/C1 = \frac{1.771 \times 10^{-9}}{106 \times 10^{-12}}$$

= 16.7 V

Q 2.10) A 12pF capacitor is connected to a 50V battery. How much electrostatic energy is stored in the capacitor?

Soln.: Given,

Capacitance of the capacitor, C = 12pF = 12×10^{-12} F

Potential difference, V = 50 V

Electrostatic energy stored in the capacitor is given by the relation,

$$E = \frac{1}{2} CV^2 = \frac{1}{2} \times 12 \times 10^{-12} \times (50)^2 \text{ J} = 1.5 \times 10^{-8} \text{ J}$$

Therefore, the electrostatic energy stored in the capacitor is 1.5×10^{-8} J.

was disconnected.

Q 2.11) A 600pF capacitor is charged by a 200V supply. It is then disconnected from the supply and is connected to another uncharged 600 pF capacitor. How much electrostatic energy is lost in the process?

Soln.: Given,

Capacitance, C = 600pF

Potential difference, V = 200v

Electrostatic energy stored in the capacitor is given by :

$$E_1 = \frac{1}{2} CV^2 = \frac{1}{2} \times (600 \times 10^{-12}) \times (200)^2 \text{J} = 1.2 \times 10^{-5} \text{ J}$$

Acc. to the question, the source is disconnected to the 600pF and connected to another capacitor of 600pF, then equivalent capacitance (C_{eq}) of the combination is given by,

$$\frac{1}{C_{eq}} = \frac{1}{C} + \frac{1}{C} \quad \frac{1}{C_{eq}} = \frac{1}{600} + \frac{1}{600}$$

$$= \frac{2}{600} = \frac{1}{300}$$

C_{eq} = 300pF

New electrostatic energy can be calculated by:

$$E_2 = \frac{1}{2} CV^2 = \frac{1}{2} \times 300 \times (200)^2 \text{ J} = 0.6 \times 10^{-5} \text{ J}$$

Loss in electrostatic energy,

$E = E_1 - E_2$

$E = 1.2 \times 10^{-5} - 0.6 \times 10^{-5}$ J $= 0.6 \times 10^{-5}$ J $= 6 \times 10^{-6}$ J

Therefore, the electrostatic energy lost in the process is 6×10^{-6} J.